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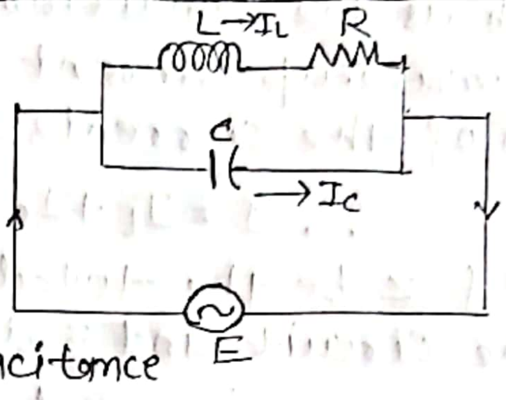
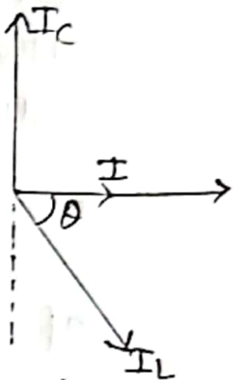
Expression for current in parallel L.C.R Circuit.





A parallel circuit consists of an inductance L with a resistance R in parallel with a capacitance C as shown in figure. Resistance associated with capacitance is assumed to be negligible. An A.C Source of e.m.f E is connected across this combination. parallel resonance is also called anti-resonance because the impedance is maximum at resonance whereas in the Series Circuit, the impedance is minimum at resonance.

Let I be the instantaneous current flowing in-to the circuit at an instant t and I_L and I_c be the corresponding currents through the inductance and capacitance. Since I_L lags behind the applied e.m.f. by 90° and I_c leads the applied e.m.f. by 90°. I_L and I_c will almost be out of phase if R is small, as show in the vector diagram. If we resolve I_L into two components are balances I_c at resonance the other I, known as make up current, is in phase with the applied e.m.f. E. The phase angle θ between I and E is given by $\tan\theta = \frac{\omega L}{R}$. As R is very small

θ is large and hence I is small. The impedence of the circuit is very large and will be infinite for $R = 0$ and then $\theta = 90°$. Therefore, there is a small current in phase with the applied e.m.f. which is the condition for resonance. In this case the make-up current I will be equal to the vector sum of the currents I_L and I_C

$$\therefore I = I_L + I_C \quad \text{——— (1)}$$

If z be the total complex vector impedence of the circuit and E is applied e.m.f. then.

$$I = \frac{E}{Z} \text{ and } I_L = \frac{E}{R + j\omega L}$$

$$\text{and } I_C = \frac{E}{1/j\omega C} = j\omega CE$$

$$\therefore \frac{E}{Z} = \frac{E}{R + j\omega L} + j\omega CE$$

$$\text{or, } \frac{1}{Z} = \frac{1}{R + j\omega L} + j\omega C = \frac{R - j\omega L}{R^2 + \omega^2 L^2} + j\omega C$$

$$\text{or, } \frac{1}{Z} = \frac{1}{R^2 + \omega^2 L^2} + j\left(\frac{\omega C R^2 + \omega^3 L^2 C - \omega L}{(R^2 + \omega^2 L^2)}\right) \quad -(2)$$

This give complex vector admittance of circuit. The magnitude of the admittance is

$$\gamma = \left|\frac{1}{Z}\right| = \frac{\sqrt{R^2 + (\omega C R^2 + \omega^3 L^2 C - \omega L)^2}}{R^2 + \omega^2 L^2} \quad (3)$$

The admitance will be minimum or the impedance will be maximum when

$$\omega C R^2 + \omega^2 L^2 C - \omega L = 0$$

This means that the reactive components of γ must be zero. The circuit current given by $I = E/Z$ will then be minimum for a given supply e.m.f. This occurs at resonance. The frequency at resonance is called the resonant frequency f_R. Thus, at resonance

$$\omega C R^2 + \omega^3 L^2 C - \omega L = 0 \quad , \text{ where } \omega = 2\pi f_R$$

$$\therefore \omega = 2\pi f_R = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \quad \text{———} (4)$$

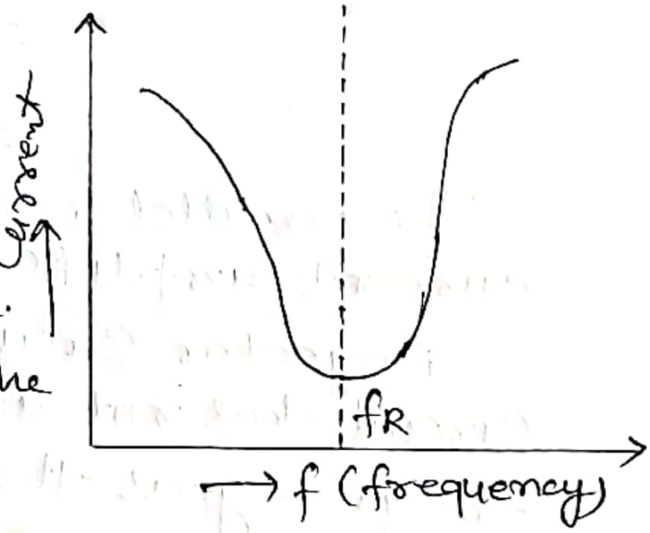
$$\text{or, } f_R = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \quad \text{———} (5)$$

for parallel resonance to occur, $\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$ should be real

$$\therefore \frac{1}{LC} - \frac{R^2}{L^2} > 0, \text{ or, } R < \sqrt{\frac{L}{C}}$$

If $R < \sqrt{L/C}$, no parallel resonance can occur. Hence the resistance R should be kept as low as possible. If $R \longrightarrow 0$, $f_R = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$, which is the frequency at series resonance which is the usual case.

The variation of current with frequency in parallel resonance circuit is shown in figure. The curve shows that the current is minimum at resonance frequency.



The impedance at resonance is called dynamic resistance, which is the resiprocal of the real part equation (2)

$$\therefore Z = \frac{R^2 + \omega^2 L^2}{R}$$

$$= R + \frac{L^2}{R}\left(\frac{1}{LC} - \frac{R^2}{L^2}\right) = \frac{L}{RC} \quad \text{(from eqn (4))}$$

The peak current from the supply at resonance known as make-up current is given by

$$I = \frac{E_0}{L/CR}$$

$$= \frac{E_0 CR}{L}$$

where E_0 is peak value of e.m.f

The Current branch Capacitance branch

$$I_C = \frac{E_0}{1/\omega C}$$

$$= E_0 \omega C$$

At resonance $I_C = I_L$ for small R.

$$\therefore I_C/I = \frac{E_0 \omega C \times L}{E_0 CR}$$

$$= \frac{\omega L}{R}$$

$= Q$, the current magnification

The parallel resonant circuit therefore gives current amplification of $\omega L/R$

Rejector Circuit:— The parallel resonant circuit does not allow the current of the same frequency as the natural frequency of circuit. Thus it can be used to suppress the current of this particular frequency out of the currents of many other frequencies. Hence a circuit is called a 'Rejector' or 'filter' circuit.

The parallel resonant circuit is widely used in radio due its high impedance at resonance and the current magnification which it can give.